CHEMISTRY - 1999

PART - A

Directions : *Select the most appropriate alternative A, B, C or D in questions 1–25.*

1. The electrons, identified by quantum numbers n and l, (i) $n = 4, l = 1$, (ii) $n = 4, l = 0$, (iii) $n = 3, l = 2$, and (iv) $n = 3, l = 1$ can be placed in order of increasing energy, from the lowest to highest, as :

(A) (iv) < (ii) < (iii) < (i)
(B) (ii) < (iv) < (i) < (iii)
(C) (i) < (iii) < (ii) < (iv)
(D) (iii) < (i) < (iv) < (ii)

2. The number of neutrons accompanying the formation of $^{139}_{54}Xe$ and $^{94}_{38}Sr$ from the absorption of a slow neutron by $^{235}_{92}U$, followed by nuclear fission is :

(A) 0
(B) 2
(C) 1
(D) 3

3. The correct order of increasing C—O bond length of CO, CO_3^{2-}, CO_2 is :

(A) $CO_3^{2-} < CO_2 < CO$
(B) $CO_2 < CO_3^{2-} < CO$
(C) $CO < CO_3^{2-} < CO_2$
(D) $CO < CO_2 < CO_3^{2-}$

4. A gas will approach ideal behaviour at :

(A) low temperature and low pressure
(B) low temperature and high pressure
(C) high temperature and low pressure
(D) high temperature and high pressure

5. The normality of 0.3 M phosphorus acid (H_3PO_3) is :

(A) 0.1
(B) 0.9
(C) 0.3
(D) 0.6

6. The coordination number of a metal crystallizing in a hexagonal close-packed structure is :

(A) 12
(B) 4
(C) 8
(D) 6

7. A gas X at 1 atm is bubbled through a solution containing a mixture of 1 M Y^- and 1 M Z^- at 25°C. If the reduction potential of Z > Y > X, then :

(A) Y will oxidize X and not Z
(B) Y will oxidize Z and not X
(C) Y will oxidize both X and Z
(D) Y will reduce both X and Z.

8. The pH of 0.1 M solution of the following salts increases in the order :

(A) NaCl < NH_4Cl < NaCN < HCl
(B) HCl < NH_4Cl < NaCl < NaCN
(C) NaCN < NH_4Cl < NaCl < HCl
(D) HCl < NaCl < NaCN < NH_4Cl

9. For the chemical reaction $3X(g) + Y(g) \rightleftharpoons X_3Y(g)$, the amount of X_3Y at equilibrium is affected by :
(A) temperature and pressure
(B) temperature only
(C) pressure only
(D) temperature, pressure and catalyst

10. In the dichromate dianion :
(A) 4 Cr—O bonds are equivalent
(B) 6 Cr—O bonds are equivalent
(C) all Cr—O bonds are equivalent
(D) all Cr—O bonds are nonequivalent

11. One mole of calcium phosphide on reaction with excess water gives :
(A) one mole of phosphine
(B) two moles of phosphoric acid
(C) two moles of phosphine
(D) one mole of phosphorus pentoxide

12. The oxidation number of sulphur in S_8, S_2F_2, H_2S respectively, are :
(A) 0, +1 and −2
(B) +2, +1 and −2
(C) 0, +1 and +2
(D) −2, +1 and −2

13. On heating ammonium dichromate, the gas evolved is :

(A) oxygen
(B) ammonia
(C) nitrous oxide
(D) nitrogen

14. In the commerical electrochemical process for aluminium extraction, the electrolyte used is :
(A) $Al(OH)_3$ in NaOH solution
(B) an aqueous solution of $Al_2(SO_4)_3$
(C) a molten mixture of Al_2O_3 and Na_3AlF_6
(D) a molten mixture of AlO(OH) and $Al(OH)_3$

15. The geometry of H_2S and its dipole moment are :
(A) angular and non-zero
(B) angular and zero
(C) linear and non-zero
(D) linear and zero

16. The geometry of $Ni(CO)_4$ and $Ni(PPh_3)_2Cl_2$ are :
(A) both square planar
(B) tetrahedral and square planar, respectively
(C) both tetrahedral
(D) square planar and tetrahedral, respectively

17. In compounds of type ECl_3, where E = B, P, As or Bi, the angles Cl—E—Cl for different E are in the order :
(A) B > P = As = Bi
(B) B > P > As > Bi
(C) B < P = As = Bi
(D) B < P < As < Bi

18. In the compound $CH_2 = CH—CH_2—CH_2—C \equiv CH$, the $C_2 - C_3$ bond is of the type :

(A) $sp - sp^2$ (B) $sp^3 - sp^3$

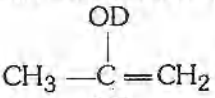
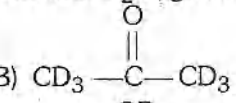
(C) $sp - sp^3$ (D) $sp^2 - sp^3$

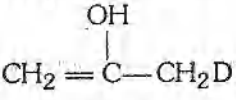
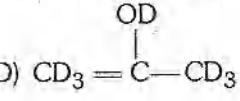
19. When propionic acid is treated with aqueous sodium bicarbonate, CO_2 is liberated. The 'C' of CO_2 comes from :

(A) methyl group (B) carboxylic acid group

(C) methylene group (D) bicarbonate

20. The enol form of acetone, after treatment with D_2O, gives :

(A)  (B) 

(C)  (D) 

21. A positive carbylamine test is given by :

(A) N, N–dimethylaniline (B) 2, 4-dimethylaniline

(C) N-methyl-o-methylaniline (D) *p*-methylbenzylamine

22. The optically active tartaric acid is named as D – (+) – tartaric acid because it has a positive :

(A) optical rotation and is derived from D-glucose

(B) pH in organic solvent

(C) optical rotation and is derived from D – (+) – glyceraldehyde

(D) optical rotation only when substituted by deuterium

23. A solution of (+) –2–chloro–2–phenylethane in toluene racemises slowly in the presence of small amount of $SbCl_5$, due to the formation of :

(A) carbanion (B) carbene

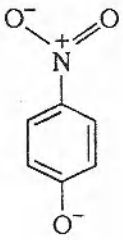
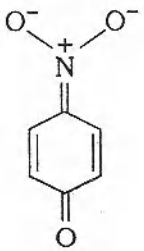
(C) free-radical (D) carbocation

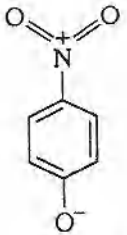
24. The product(s) obtained *via* oxymercuration ($HgSO_4 + H_2SO_4$) of 1-butyne would be :

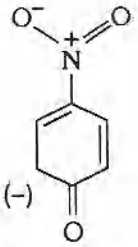
(A)  (B) $CH_3—CH_2—CH_2—CHO$

(C) $CH_3—CH_2—CHO + HCHO$ (D) $CH_3—CH_2—COOH + HCOOH$

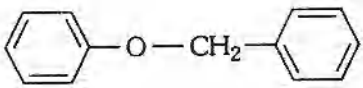
25. The most unlikely representation of resonance structures of *p*-nitrophenoxide ion is :

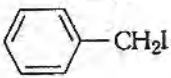
(A)  (B) 

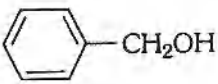
(C) 

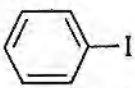
(D) 

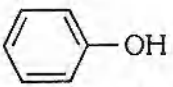
Directions : *Question numbers 26–35 carry 3 marks each and may have more than one correct answer. All correct answers must be marked to get any credit in these questions.*

26. The ether  when treated with HI produces :

(A) 

(B) 

(C) 

(D) 

27. Toluene, when treated with Br_2/Fe, gives p-bromotoluene as the major product because the CH_3 group :

(A) is *para* directing

(B) is *meta* directing

(C) activates the ring by hyperconjugation

(D) deactivates the ring

28. The following statement(s) is (are) correct :

(A) A plot of log K_p *versus* $1/T$ is linear

(B) A plot of log [X] *versus* time is linear for a first order reaction, $X \rightarrow P$

(C) A plot of log p *versus* $1/T$ is linear at constant volume

(D) A plot of p *versus* $1/V$ is linear at constant temperature

29. The following is (are) endothermic reaction(s) :

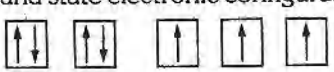
(A) Combustion of methane

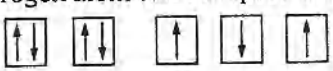
(B) Decomposition of water

(C) Dehydrogenation of ethane to ethylene

(D) Conversion of graphite to diamond.

30. Ground state electronic configuration of nitrogen atom can be represented by :

(A) 

(B) 

(C) 

(D)

31. In the depression of freezing point experiment, it is found that the :

(A) vapour pressure of the solution is less than that of pure solvent

(B) vapour pressure of the solution is more than that of pure solvent

(C) only solute molecules solidify at the freezing point
(D) only solvent molecules solidify at the freezing point

32. Ionic radii of :
(A) $Ti^{4+} < Mn^{7+}$ (B) $^{35}Cl^- < {}^{37}Cl^-$
(C) $K^+ > Cl^-$ (D) $P^{3+} > P^{5+}$

33. Ammonia, on reaction with hypochlorite anion, can form :
(A) NO (B) NH_4Cl
(C) N_2H_4 (D) HNO_2

34. A buffer solution can be prepared from a mixture of :
(A) sodium acetate and acetic acid in water
(B) sodium acetate and hydrochloric acid in water
(C) ammonia and ammonium chloride in water
(D) ammonia and sodium hydroxide in water

35. An aromatic molecule will :
(A) have $4n\ \pi$ electrons (B) have $(4n + 2)\ \pi$ electrons
(C) be planar (D) be cyclic

ANSWERS

1. (A)	**2.** (D)	**3.** (D)	**4.** (C)	**5.** (D)	**6.** (A)
7. (A)	**8.** (B)	**9.** (A)	**10.** (B)	**11.** (C)	**12.** (A)
13. (D)	**14.** (C)	**15.** (A)	**16.** (C)	**17.** (B)	**18.** (D)
19. (D)	**20.** (B)	**21.** (B), (D)	**22.** (C)	**23.** (D)	**24.** (A)
25. (C)	**26.** (A), (D)	**27.** (A), (D)	**28.** (A), (B), (D)	**29.** (B), (C) (D)	**30.** (A), (D)
31. (A), (D)	**32.** (D)	**33.** (C)	**34.** (A)	**35.** (B), (C) & (D)	

SOLUTIONS

Reason of Correctness

1. On the basis of $(n + l)$ Rule
(i) Value of $(n + l) = 4 + 1 = 5$
(ii) Value of $(n + l) = 4 + 0 = 4$
(iii) Value of $(n + l) = 3 + 2 = 5$
(iv) Value of $(n + l) = 3 + 1 = 4$
Hence correct order of energy (iv) < (ii) < (iii) < (i)

In these $(n + l)$ is lower for (ii) & (iv) but equal both, so in these n is minimum for (iv).
Hence energy order = (iv) < (ii)
Similaring in (i) & (iii)
(iii) < (i)

Ans. (A)

2. $_{92}U^{235} + {}_0n^1 \rightarrow {}_{54}Xe^{139} + {}_{38}Sr^{94} + 3{}_0n^1$ **Ans. (D)**

3. Bond length $\propto \dfrac{1}{\text{Bond order}}$

Bond order $CO_3^{2-} < CO_2 < CO$

Bond order in CO = 3 (with the help of molecular orbital theory)

Bond order in $CO_2 = \dfrac{\text{no. of bonds in all possible sides}}{\text{no. of resonating structure}}$ (By resonance)

$= \dfrac{4}{2} = 2$

Bond order in $CO_3^{2-} = \dfrac{4}{3} = 1.33$ (By resonance)

MATHEMATICS - 1999

PART - A

Directions : *Select the most appropriate alternative A, B, C or D in questions 1–25.*

1. If $i = \sqrt{-1}$, then $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$ is equal to :

(A) $1 - i\sqrt{3}$ (B) $-1 + i\sqrt{3}$
(C) $i\sqrt{3}$ (D) $-i\sqrt{3}$

2. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G. P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) :

(A) lie on a straight line (B) lie on an ellipse
(C) lie on a circle (D) are vertices of a triangle

3. If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is :

(A) $\left(\frac{1}{2}\right)^{x(x-1)}$ (B) $\frac{1}{2}(1 + \sqrt{1 + 4\log_2 x})$
(C) $\frac{1}{2}(1 - \sqrt{1 + 4\log_2 x})$ (D) not defined

4. The harmonic mean of the roots of the equation
$(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$ is :

(A) 2 (B) 4
(C) 6 (D) 8

5. The function $f(x) = \sin^4 x + \cos^4 x$ increases if :

(A) $0 < x < \frac{\pi}{8}$ (B) $\frac{\pi}{4} < x < \frac{3\pi}{8}$
(C) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ (D) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

6. The curve described parametrically by $x = t^2 + t + 1$, $y = t^2 - t + 1$ represents :

(A) a pair of straight lines (B) an ellipse
(C) a parabola (D) a hyperbola

7. In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2 + bx + c = 0$ $(a \neq 0)$, then :

(A) $a + b = c$ (B) $b + c = a$
(C) $a + c = b$ (D) $b = c$

8. If for a real number y, $[y]$ is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2} [2 \sin x]\, dx$ is :

(A) $-\pi$ (B) 0

(C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

9. Let $a_1, a_2, \ldots, a_{10}$ be in A. P. and $h_1, h_2, \ldots, h_{10}$ be in H. P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is :

(A) 2 (B) 3

(C) 5 (D) 6

10. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is 30°, then $\left|(\vec{a} \times \vec{b}) \times \vec{c}\right| =$

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$

(C) 2 (D) 3

11. The number of real solutions of

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2} \text{ is :}$$

(A) zero (B) one

(C) two (D) infinite.

12. Let $P(a \sec\theta, b \tan\theta)$ and $Q(a \sec\phi, b \tan\phi)$, where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If (h, k) is the point of intersection of the normals at P and Q, then k is equal to :

(A) $\frac{a^2 + b^2}{a}$ (B) $-\left(\frac{a^2 + b^2}{a}\right)$

(C) $\frac{a^2 + b^2}{b}$ (D) $-\left(\frac{a^2 + b^2}{b}\right)$

13. Let PQR be a right angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is :

(A) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$

(B) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

(C) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$

(D) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

14. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$ then $f(100)$ is equal to :

(A) 0 (B) 1

(C) 100 (D) -100

15. The function $f(x) = [x]^2 - [x^2]$ (where [y] is the greatest integer less than or equal to y), is discontinuous at :

(A) all integers (B) all integers except 0 and 1

(C) all integers except 0 (D) all integers except 1

16. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x-axis, then :

(A) $p^2 = q^2$ (B) $p^2 = 8q^2$

(C) $p^2 < 8q^2$ (D) $p^2 > 8q^2$

17. The function $f(x) = (x^2 - 1)\left|x^2 - 3x + 2\right| + \cos(|x|)$ is NOT differentiable at :

(A) -1 (B) 0

(C) 1 (D) 2

18. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then :

(A) $a < 2$ (B) $2 \leq a \leq 3$

(C) $3 < a \leq 4$ (D) $a > 4$

19. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is :

(A) $y = 2$ (B) $y = 2x$

(C) $y = 2x - 4$ (D) $y = 2x^2 - 4$

20. $\lim\limits_{x \to 0} \frac{x\tan 2x - 2x\tan x}{(1 - \cos 2x)^2}$ is :

(A) 2 (B) -2

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

21. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c}$ =

(A) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$ (B) $\frac{1}{\sqrt{3}}(-\hat{i} - \hat{j} - \hat{k})$

(C) $\frac{1}{\sqrt{5}}(\hat{i} - 2\hat{j})$ (D) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

22. If in the expansion of $(1 + x)^m (1 - x)^n$, the coefficients of x and x^2 are 3 and -6 respectively, then m is :

(A) 6 (B) 9

(C) 12 (D) 24

23. $\int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x}$ is equal to :

(A) 2 (B) -2

(C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

24. If $x = 9$ is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangents is :

(A) $9x^2 - 8y^2 + 18x - 9 = 0$

(B) $9x^2 - 8y^2 - 18x + 9 = 0$

(C) $9x^2 - 8y^2 - 18x - 9 = 0$

(D) $9x^2 - 8y^2 + 18x + 9 = 0$

25. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals :

(A) $\frac{1}{4}$ (B) $\frac{1}{7}$

(C) $\frac{1}{8}$ (D) $\frac{1}{49}$

Directions : *Question numbers 26–35 carry 3 marks each and may have more than one correct answers. All correct answers must be marked to get any credit in these questions :*

26. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1 ?

(A) $x + y = 0$ (B) $x - y = 0$

(C) $x + 7y = 0$ (D) $x - 7y = 0$

27. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - (\vec{a} \cdot \vec{b})\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $|\vec{v}|$ is :

(A) $\left|\vec{u}\right|$ (B) $\left|\vec{u}\right| + \left|\vec{u} \cdot \vec{a}\right|$

(C) $\left|\vec{u}\right| + \left|\vec{u} \cdot \vec{b}\right|$ (D) $\left|\vec{u}\right| + \vec{u} \cdot (\vec{a} + \vec{b})$

28. For a positive integer n, let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots + \frac{1}{(2^n) - 1}$. Then :

(A) $a(100) \leq 100$ (B) $a(100) > (100)$

(C) $a(200) \leq 100$ (D) $a(200) > 100$

29. The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3 (t-3)^5 dt$ has a local minimum at $x =$

(A) 0 (B) 1

(C) 2 (D) 3

30. On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are :

(A) $\left(\frac{2}{5}, \frac{1}{5}\right)$ (B) $\left(-\frac{2}{5}, \frac{1}{5}\right)$

(C) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (D) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

31. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c, respectively. Of these subjects, the student has a 75% chance of passing in atleast one, a 50% chance of passing in atleast two, and a 40% chance of passing in exactly two. Which of the following relations are true ?

(A) $p + m + c = \frac{19}{20}$ (B) $p + m + c = \frac{27}{20}$

(C) $pmc = \frac{1}{10}$ (D) $pmc = \frac{1}{4}$

32. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of :

(A) order 1 (B) order 2

(C) degree 3 (D) degree 4

33. Let $S_1, S_2 \ldots$ be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm, then for which of the following values of n is the area of S_n less than 1sq. cm?

(A) 7 (B) 8

(C) 9 (D) 10

34. For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals $\frac{9}{2}$?

(A) –4 (B) –2

(C) 2 (D) 4

35. For a positive integer n, let

$f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\ldots(1+\sec 2^n\theta)$. Then

(A) $f_2\left(\frac{\pi}{16}\right) = 1$ (B) $f_3\left(\frac{\pi}{32}\right) = 1$

(C) $f_4\left(\frac{\pi}{64}\right) = 1$ (D) $f_5\left(\frac{\pi}{128}\right) = 1$

ANSWERS

1. (C)	**2.** (A)	**3.** (B)	**4.** (B)	**5.** (B)	**6.** (C)
7. (A)	**8.** (C)	**9.** (D)	**10.** (B)	**11.** (C)	**12.** (D)
13. (B)	**14.** (A)	**15.** (B)	**16.** (D)	**17.** (D)	**18.** (A)
19. (C)	**20.** (C)	**21.** (A)	**22.** (C)	**23.** (A)	**24.** (B)
25. (A)	**26.** (A), (C)	**27.** (B), (C)	**28.** (A), (D)	**29.** (B), (D)	**30.** (B), (D)
31. (B),	**32.** (A), (C)	**33.** (B), (C), (D)	**34.** (B), (D)	**35.** (A), (B), (C), (D).	

SOLUTIONS

1. **Imp. note :** If in a complex no. $a + ib$, the ratio $a : b$ is $1 : \sqrt{3}$ then always try to convert that complex no. in ω.

Here $\omega = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

Therefore, $4 + 5\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{334} + 3\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{365}$

$= 4 + 5\omega^{334} + 3\omega^{365}$

$= 4 + 5\cdot(\omega^3)^{111}\cdot\omega + 3\cdot(\omega^3)^{123}\cdot\omega^2$

$= 4 + 5\omega + 3\omega^2$ $\because$ $\omega^3 = 1$

$= 1 + 3 + 2\omega + 3\omega + 3\omega^2$

$= 1 + 2\omega + 3(1 + \omega + \omega^2) = 1 + 2\omega + 3\times 0$ $\because 1 + \omega + \omega^2 = 0$

$= 1 + (-1 + \sqrt{3}i) = \sqrt{3}i$ Therefore, (C) is the answer.

2. Let $\frac{x_2}{x_1} = \frac{x_3}{x_2} = r$ and $\frac{y_2}{y_1} = \frac{y_3}{y_2} = r$

$\Rightarrow x_2 = x_1 r$, $x_3 = x_1 r^2$ and $y_2 = y_1 r$ and $y_3 = yr^2$.

again, $\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 1 \\ x_1 r & y_1 r & 1 \\ x_1 r^2 & y_1 r^2 & 1 \end{vmatrix}$

PHYSICS - 1999

PART - A

Directions : *Select the most appropriate alternative a, b, c & d in questions 1-25*

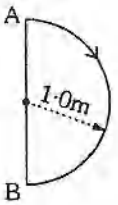
1. A closed compartment containing gas is moving with some acceleration in horizontal direction. Neglect effect of gravity. Then the pressure in the compartment is :

(A) same everywhere (B) lower in front side
(C) lower in rear side (D) lower in upper side.

2. The ratio of the speed of sound in nitrogen gas to that in helium gas at 300K is :

(A) $\sqrt{2/7}$ (B) $\sqrt{1/7}$
(C) $\sqrt{3}/5$ (D) $\sqrt{6}/5$

3. In 1·0S, a particle goes from point A to point B, moving in a semicircle (see figure). The magnitude of the average velocity is :

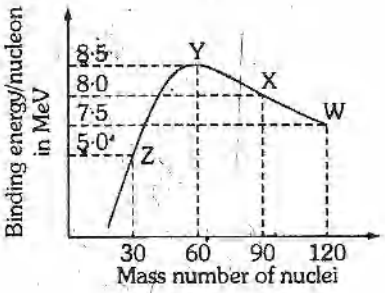


(A) 3·14 m/s (B) 2·0 m/s
(C) 1·0 m/s (D) zero

4. A charged particle is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The particle will move in a :

(A) straight line (B) circle
(C) helix (D) cycloid

5. Binding energy per nucleon Vs mass number curve for nuclei is shown in figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is :



(A) $Y \rightarrow 2Z$
(B) $W \rightarrow X + Z$
(C) $W \rightarrow 2Y$
(D) $X \rightarrow Y + Z$

6. Order of magnitude of density of uranium nucleus is ($m_p = 1.67 \times 10^{-27}$ kg) :

(A) 10^{20} kg/m^3 (B) 10^{17} kg/m^3
(C) 10^{14} kg/m^3 (D) 10^{11} kg/m^3

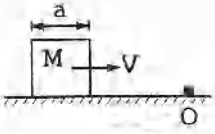
7. Two identical circular loops of metal wire are lying on a table without touching each other. Loop A carries a current which increases with time. In response, the loop B :

(A) remains stastionery
(B) is attracted by the loop A
(C) is repelled by the loop A
(D) rotates about its CM, with CM fixed.

8. A spring of force constant K is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of :
(A) 2/3 K
(B) 3/2 K
(C) 3 K
(D) 6 K

9. ^{22}Ne nucleus, after absorbing energy, decays into two α-particles and an unknown nucleus. The unknown nucleus is :
(A) nitrogen
(B) carbon
(C) boron
(D) oxygen.

10. A cubical block of side a moving with velocity V on a horizontal smooth plane as shown. It hits a ridge at point O. The angular speed of the block after it hits O is :

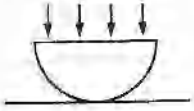


(A) 3V/4a
(B) 3V/2a
(C) $\sqrt{3}V/\sqrt{2}a$
(D) zero

11. Yellow light is used in a single slit diffraction experiment with slit width of 0·6 mm. If yellow light is replaced by X-rays, then the observed pattern will reveal :
(A) that the central maximum is narrower
(B) more number of fringes
(C) less number of fringes
(D) no diffraction pattern

12. Two identical metal plates are given positive charges Q_1 and Q_2 $(< Q_1)$ respectively. If they are now brought close together to form a parallel plate capacitor with capacitance C, the potential difference between them is :
(A) $(Q_1 + Q_2)/2C$
(B) $(Q_1 + Q_2)/C$
(C) $(Q_1 - Q_2)/C$
(D) $(Q_1 - Q_2)/2C$

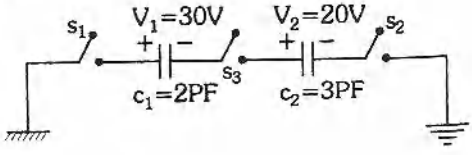
13. A thin slice is cut out of a glass cylinder along a plane parallel to its axis. The slice is placed on a flat plate as shown. The observed interference fringes from this combination shall be :



(A) straight
(B) circular
(C) equally spaced
(D) having fringe spacing which increases as we go outwards.

14. A coil of inductance 8.4 mH and resistance 6Ω is connected to a 12V battery. The current in the coil is 1.0A at approximately the time :
(A) 500 s
(B) 20 s
(C) 35 ms
(D) 1 ms

15. For the circuit shown, which of the following statements is true :



(A) With S_1 closed,
$V_1 = 15V, V_2 = 20V$
(B) With S_3 closed,
$V_1 = V_2 = 25V$
(C) With S_1 and S_2 closed, $V_1 = V_2 = 0$
(D) With S_1 and S_3 closed, $V_1 = 30V$, $V_2 = 20V$

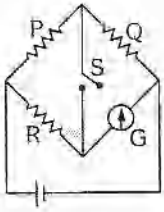
16. A concave lens of glass, refractive index 1.5 has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a :

(A) Convergent lens of focal length 3.5 R
(B) Convergent lens of focal length 3.0 R
(C) divergent lens of focal length 3.5 R
(D) divergent lens of focal length 3.0 R

17. A gas mixture consists of 2 moles of oxygen and 4 moles of argon at temperature T. Neglecting all vibrational modes, the total internal energy of the system is :

(A) 4 RT (B) 15 RT
(C) 9 RT (D) 11 RT

18. In the circuit shown $P \neq R$, the reading of galvonometer is same with switch S open or closed. Then :



(A) $I_R = I_G$
(B) $I_P = I_G$
(C) $I_Q = I_G$
(D) $I_Q = I_R$

19. A smooth sphere A is moving on a frictionless horizontal plane with angular velocity ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B respectively. Then :

(A) $\omega_A < \omega_B$ (B) $\omega_A = \omega_B$
(C) $\omega_A = \omega$ (D) $\omega_B = \omega$

20. In hydrogen spectrum the wavelength of H_α line is 656 nm; whereas in the spectrum of a distant galaxy H_α line wavelength is 706 nm. Estimated speed of galaxy with respect to earth is :

(A) 2×10^8 m/s (B) 2×10^7 m/s
(C) 2×10^6 m/s (D) 2×10^5 m/s

21. A particle free to move along the x-axis has potential energy given by $U(x) = K[1 - \exp(-x^2)]$ for $-\infty \leq x \leq +\infty$ where K is a positive constant of appropriate dimensions. Then :

(A) At points away from the origin, the particle is in unstable equilibrium
(B) For any finite non-zero value of x, there is a force directed away from the origin
(C) If its total mechanical energy is K/2, it has its minimum kinetic energy at the origin.
(D) For small displacements from $x = 0$, the motion is simple hormonic

22. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non-zero velocities. The ratio of the de-Broglie wavelengths of the particles λ_1/λ_2 is :

(A) m_1/m_2 (B) m_2/m_1
(C) 1.0 (D) $\sqrt{m_2}/\sqrt{m_1}$

23. A circular loop of radius R, carrying current I, lies in x-y plane with its centre at the origin. The total magnetic flux through x-y plane is :

(A) directly proportional to I (B) directly proportional to R

(C) directly proportional to R^2 (D) zero

24. Which of the following is a correct statement :

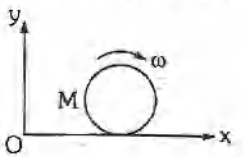
(A) Beta rays are same as cathode rays

(B) Gamma rays are high energy neutrons

(C) Alpha particles are singly ionized helium atoms

(D) Protons and neutrons have exactly the same mass.

25. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown. The magnitude of angular momentum of the disc about the origin O is :



(A) $\left(\frac{1}{2}\right) MR^2\omega$ (B) $MR^2\omega$

(C) $\left(\frac{3}{2}\right) MR^2\omega$ (D) $2\,MR^2\omega$

Directions : *Question numbers 26–35 carry 3 marks each and may have more than one correct answers. All correct answers must be marked to get any credit in these questions.*

26. The coordinates of a particle moving in a plane are given by x (t) = a cos(pt) and y (t) = b sin (pt) where a, b (< a) and p are positive constants of appropriate dimensions. Then :

(A) the path of the particle is an ellipse

(B) the velocity and acceleration of the particle are normal to each other at $t = \pi/2p$

(C) the acceleration of the particle is always directed towards a focus

(D) the distance travelled by the particle in time interval t = 0 to $t = \pi/2p$ is a.

27. The half-life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially both of them have the same number of atoms. Then :

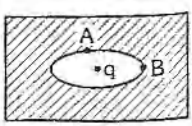
(A) X and Y have the same decay rate initially

(B) X and Y decay at the same rate always

(C) Y will decay at a faster rate than X

(D) X will decay at faster rate than Y

28. An elliptical cavity is carved within a perfect conductor. A positive charge q is placed at the centre of the cavity. The points A and B are on the cavity surface as shown in the figure. Then :



(A) electric field near A in the cavity = electric field near B in the cavity

(B) charge density at A = charge density at B

(C) potential at A = potential at B

(D) total electric field flux through the surface of the cavity is q/ϵ_0.

29. Three simple harmonic motions in the same direction having the same amplitude and same period are superposed. If each differ in phase from the next by 45°, then :

(A) the resultant amplitude is $(1 + \sqrt{2})\ a$

(B) the phase of the resultant motion relative to the first is 90°

(C) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion

(D) the resulting motion is not simple harmonic

30. As a wave propagates :

(A) the wave intensity remains constant for a plane wave

(B) the wave intensity decreases as the inverse of the distance from the source for a spherical wave

(C) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave

(D) total intensity of the spherical wave over the spherical surface centered at the source remains constant at all times.

31. A bimetallic strip is formed out of two identical strips – one of copper and the other of brass. The coefficients of linear expansion of the two metals are α_C and α_B. On heating, the temperature of the strip goes up by ΔT and the strip bends to form an arc of radius of curvature R. Then R is :

(A) proportional to ΔT

(B) inversely proportional to ΔT

(C) proportional to $|\alpha_B - \alpha_C|$

(D) inversely proportional to $|\alpha_B - \alpha_C|$

32. When a potential difference is applied across, the current passing through :

(A) an insulater at 0 K is zero

(B) a semiconductor at 0 K is zero

(C) a metal at 0 K is finite

(D) a p-n diode at 300 K is finite if it is reverse biased.

33. $Y(x, t) = \dfrac{0.8}{[(4x + 5t)^2 + 5]}$ represents a moving pulse where x and y are in metres and t in second. Then :

(A) pulse is moving in positive x direction

(B) in 2s it will travel a distance of 2.5 m

(C) its maximum displacement is 0.16 m

(D) it is a symmetric pulse

34. In a wave motion $y = a \sin(Kx - \omega t)$, y can represent :

(A) electric field (B) magnetic field

(C) displacement (D) pressure.

35. Standing waves can be produced :

(A) on a string clamped at both ends

(B) on a string clamped at one end and free at the other

(C) when incident wave gets reflected from a wall

(D) when two identical waves with a phase difference of π are moving in the same direction.

ANSWERS

1. B,	**2.** C,	**3.** B,	**4.** A,	**5.** C,	**6.** B,
7. C,	**8.** B,	**9.** B,	**10.** A,	**11.** D,	**12.** D,
13. A,	**14.** D,	**15.** D,	**16.** A,	**17.** D,	**18.** A,
19. C,	**20.** B,	**21.** D,	**22.** C,	**23.** D,	**24.** A,
25. C,	**26.** A, B, C,	**27.** C,	**28.** C, D,	**29.** A, C,	**30.** A, C, D,
31. B, D,	**32.** A, B, D,	**33.** B, C, D,	**34.** A, B, C, D,		**35.** A, B, C.

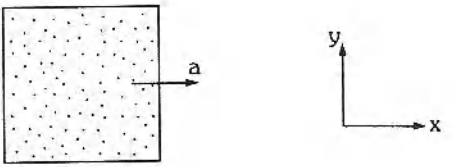
SOLUTIONS

1. (B)

If a fluid (gas or liquid) is accelerated in positive x-direction, then pressure decreases in positive x-direction. Change in pressure has following differential equation—

$$\frac{dP}{dx} = -\rho a$$

where ρ is the density of the fluid. Therefore, pressure is lower in front side.



2. (C)

Speed of sound in an ideal gas is given by

$$V = \sqrt{\frac{\gamma RT}{M}}$$

$$\therefore \quad V \propto \sqrt{\frac{\gamma}{M}} \qquad \text{[T is same for both the gases]}$$

$$\therefore \quad \frac{V_{N_2}}{V_{He}} = \sqrt{\frac{\gamma_{N_2}}{\gamma_{H_e}} \cdot \frac{M_{H_e}}{M_{N_2}}}$$

$$= \sqrt{\frac{(7/5)}{(5/3)}\left(\frac{4}{28}\right)}$$

$$= \sqrt{3}/5$$

$\gamma_{N_2} = 7/5$ (Diatomic)

$\gamma_{He} = 5/3$ (Monoatomic)

3. (B)

$$|\text{ average velocity }| = \left|\frac{\text{Displacement}}{\text{time}}\right| = \frac{AB}{\text{time}} = \frac{2}{1} = 2\text{m/s}$$

4. (A)

The charged particle will be accelerated parallel (if it is a positive charge) or antiparallel (if it is a negative charge) to the electric field, i.e., the charged particle will move parallel or antiparallel to electric and magnetic field. Therefore net magnetic force on it will be zero and its path will be a straight line.