CHEMISTRY - 1998

PART - A

1. *Read questions 1 to 28 carefully and choose from amongst the alternatives given below each question the correct lettered choice (s). A question may have one or more correct alternatives. In order to secure any marks for a given question, all correct lettered alternative(s) must be chosen.*

1. Which of the following statement(s) is(are) correct when a mixture of NaCl and $K_2Cr_2O_7$ is gently warmed with conc. H_2SO_4 :

(A) A deep red vapour is evolved.
(B) The vapour when passed into NaOH solution gives a yellow solution of Na_2CrO_4
(C) Chlorine gas is evolved
(D) Chromyl chloride is formed.

2. Highly pure dilute solution of sodium in liquid ammonia :

(A) shows blue colour
(B) exhibits electrical conductivity
(C) produces sodium amide
(D) produces hydrogen gas

3. The reaction $CH_3CH=CH-C_6H_4-OH$ with HBr gives :

(A) $CH_3CHBrCH_2-C_6H_4-OH$
(B) $CH_3CH_2CHBr-C_6H_4-OH$
(C) $CH_3CHBrCH_2-C_6H_4-Br$
(D) $CH_3CH_2CHBr-C_6H_4-Br$

4. p-Chloroaniline and anilinium hydrochloride can be distinguished by :

(A) Sandmeyer reaction
(B) $NaHCO_3$
(C) $AgNO_3$
(D) Carbylamine test

5. The energy of an electron in the first Bohr orbit of H atom is –13.6 eV. The possible energy value(s) of the excited state(s) for electrons in Bohr orbits of hydrogen is(are) :

(A) –3.4 eV
(B) –4.2 eV
(C) –6.8 eV
(D) + 6.8 eV

6. In nitroprusside ion the iron and NO exist as Fe^{II} and NO^+ rather than Fe^{III} and NO. These forms can be differentiated by :

(A) estimating the concentration of iron.
(B) measuring the concentration of CN.
(C) measuring the solid state magnetic moment.
(D) thermally decomposing the compound.

7. Which of the following statement(s) is(are) correct :

(A) The coordination number of each type of ion in CsCl crystal is 8.

(B) A metal that crystallizes in bcc structure has a coordination number of 12.

(C) A unit cell of an ionic crystal shares some of its ions with other unit cells.

(D) The length of the unit cell in NaCl is 552 pm. (r_{Na^+} = 95 pm; r_{Cl^-} = 181 pm)

8. Sodium nitrate decomposes above –800°C to give :

(A) N_2 (B) O_2

(C) NO_2 (D) Na_2O

9. Which of the following statement(s) is (are) correct with reference to the ferrous and ferric ions :

(A) Fe^{3+} gives brown colour with potassium ferricyanide.

(B) Fe^{2+} gives blue precipitate with potassium ferricyanide.

(C) Fe^{3+} gives red colour with potassium thiocyanate.

(D) Fe^{2+} gives brown colour with ammonium thiocyanate.

10. Which of the following statement(s) is(are) correct :

(A) The electronic configuration of Cr is [Ar] $3d^5 4s^1$. (Atomic Number of Cr = 24).

(B) The magnetic quantum number may have a negative value.

(C) In silver atom, 23 electrons have a spin of one type and 24 of the opposite type. (Atomic Number of Ag = 47)

(D) The oxidation state of nitrogen in HN_3 is –3.

11. A new carbon-carbon bond formation is possible in :

(A) Cannizzaro reaction (B) Friedel-Crafts alkylation

(C) Clemmensen reduction (D) Reimer-Tiemann reaction

12. White phosphorus (P_4) has :

(A) six P–P single bonds (B) four P–P single bonds

(C) four lone pairs of electrons (D) PPP angle of 60°.

13. Which of the following will react with water :

(A) $CHCl_3$ (B) Cl_3CCHO

(C) CCl_4 (D) $ClCH_2CH_2Cl$

14. The standard reduction potential values of three metallic cations, X, Y, Z are 0.52, –3.03 and –1.18V respectively. The order of reducing power of the corresponding metals is :

(A) Y > Z > X (B) X > Y > Z

(C) Z > Y > X (D) Z > X > Y

15. Among the following compounds, which will react with acetone to give a product containing > C = N – :

(A) $C_6H_5NH_2$ (B) $(CH_3)_3N$

(C) $C_6H_5NHC_6H_5$ (D) $C_6H_5NHNH_2$

16. Which of the following compounds will show geometrical isomerism :

(A) 2-butene
(B) propene
(C) 1-phenylpropene
(D) 2-methyl-2-butene

17. The geometry and the type of hybrid orbital present about the central atom in BF_3 is :

(A) linear, sp
(B) trigonal planar, sp^2
(C) tetrahedral, sp^3
(D) pyramidal, sp^3

18. Benzyl chloride ($C_6H_5CH_2Cl$) can be prepared from toluene by chlorination with :

(A) SO_2Cl_2
(B) $SOCl_2$
(C) Cl_2
(D) NaOCl

19. Which of the following will undergo aldol condensation :

(A) acetaldehyde
(B) propanaldehyde
(C) benzaldehyde
(D) trideuteroacetaldehyde

20. Addition of high proportions of manganese makes steel useful in making rails of railroads, because manganese :

(A) gives hardness to steel.
(B) helps the formation of oxides of iron.
(C) can remove oxygen and sulphur.
(D) can show highest oxidation state of +7.

21. Decrease in atomic number is observed during :

(A) alpha emission
(B) beta emission
(C) positron emission
(D) electron capture.

22. Benzenediazonium chloride on reaction with phenol in weakly basic medium gives :

(A) diphenyl ether
(B) p-hydroxyazobenzene
(C) chlorobenzene
(D) benzene

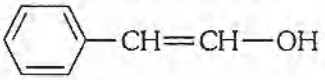
23. Among the following compounds, the strongest acid is :

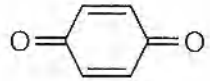
(A) $HC \equiv CH$
(B) C_6H_6
(C) C_2H_6
(D) CH_3OH

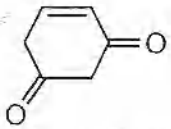
24. For a first order reaction :

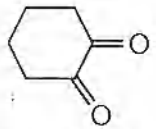
(A) the degree of dissociation is equal to $(1 - e^{-kt})$.
(B) a plot of reciprocal concentration of the reactant vs. time gives a straight line.
(C) the time taken for the completion of 75% reaction is thrice the $t_{1/2}$ of the reaction.
(D) the pre-exponential factor in the Arrhenius equation has the dimension of time, T^{-1}.

25. Tautomerism is exhibited by :

(A) 

(B) 

(C) 

(D) 

26. According to Graham's law, at a given temperature the ratio of the rates of diffusion r_A/r_B of gases A and B is given by :

(A) $(P_A/P_B)\,(M_A/M_B)^{1/2}$

(B) $(M_A/M_B)\,(P_A/P_B)^{1/2}$

(C) $(P_A/P_B)\,(M_B/M_A)^{1/2}$

(D) $(M_A/M_B)\,(P_B/P_A)^{1/2}$

(Where P and M are pressures and molecular weights of gases A and B respectively.)

27. For the reaction $CO(g) + H_2O(g) \quad CO_2(g) + H_2(g)$ at a given temperature the equilibrium amount of $CO_2(g)$ can be increased by :

(A) adding a suitable catalyst.
(B) adding an inert gas.
(C) decreasing the volume of the container.
(D) increasing the amount of CO (g).

28. Which of the following statement(s) is(are) correct :

(A) The pH of 1.0×10^{-8} M solution of HCl is 8.

(B) The conjugate base of $H_2PO_4^-$ is HPO_4^{2-}.

(C) Autoprotolysis constant of water increases with temperature.

(D) When a solution of a weak monoprotic acid is titrated against a strong base, at half-neutralisation point $pH = (1/2)\ pK_a$.

ASSERTION-REASON TYPE QUESTIONS

Directions : *The questions below (29 to 40) consist of an assertion in column 1 and the reason in column 2. Against the specific question number, write in the appropriate space.*

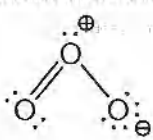
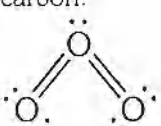
(A) If both *assertion* and *reason* are correct, and *reason* is the correct explanation of the *assertion*.

(B) If both *assertion* and *reason* are correct, but *reason* is not the correct explanation of the *assertion*.

(C) If *assertion* is correct but *reason* is incorrect.

(D) If *assertion* is incorrect but *reason* is correct.

Example :

	Assertion	Reason
	F–F bond in F_2 molecule is strong. Answer : (D)	F atom is small in size.
29.	Benzonitrile is prepared by the reaction of chlorobenzene with potassium cyanide.	Cyanide (CN^-) is a strong nucleophile.
30.	F atom has a less negative electron affinity than Cl atom.	Additional electrons are repelled more effectively by 3p electrons in Cl atom than by 2p electrons in F atom.
31.	Nuclide $^{30}_{13}Al$ is less stable than $^{40}_{20}Ca$.	Nuclides having odd number of protons and neutrons are generally unstable.
32.	Al $(OH)_3$ is amphoteric in nature.	Al—O and O—H bonds can be broken with equal ease in $Al(OH)_3$.
33.	The value of Van der Waals' constant *'a'* is larger for ammonia than for nitrogen.	Hydrogen bonding is present in ammonia.
34.	Zn^{2+} is diamagnetic.	The electrons are lost from 4s orbital to form Zn^{2+}.
35.	Addition of Br_2 to 1 –butene gives two optical isomers.	The product contains one asymmetric carbon.
36.	The electronic structure of O_3 is 	 structure is not allowed because octet around O cannot be expanded.
37.	LiCl is predominantly a covalent compound.	Electronegativity difference between Li and Cl is too small.
38.	HNO_3 is a stronger acid than HNO_2.	In HNO_3 there are two nitrogen-to-oxygen bonds whereas in HNO_2 there is only one.
39.	Sulphate is estimated as $BaSO_4$ and not as $MgSO_4$.	Ionic radius of Mg^{2+} is smaller than that of Ba^{2+}.
40.	Acetic acid does not undergo haloform reaction.	Acetic acid has no alpha hydrogens.

ANSWERS

1. (B), (C), (D)	**2.** (A), (B)	**3.** (B)	**4.** (C)	**5.** (A)	**6.** (C)
7. (A), (C), (D)	**8.** (A), (B)	**9.** (B), (C)	**10.** (A), (B), (C)	**11.** (B), (D)	**12.** (A), (C), (D)
13. (B)	**14.** (A)	**15.** (A), (D)	**16.** (A), (C)	**17.** (B)	**18.** (C)
19. (A), (B), (D)	**20.** (A), (C)	**21.** (A), (C), (D)	**22.** (B)	**23.** (D)	**24.** (A), (D)
25. (A), (C), (D)	**26.** (C)	**27.** (D)	**28.** (B), (C)	**29.** (D)	**30.** (C)
31. (C)	**32.** (A)	**33.** (A)	**34.** (B)	**35.** (A)	**36.** (A)
37. (C)	**38.** (A)	**39.** (B)	**40.** (C)		

SOLUTIONS

Reason of correctness

1. The reactions are

$$4NaCl + K_2Cr_2O_7 + 6H_2SO_4 \rightarrow 2CrO_2Cl_2 + 4NaHSO_4 + 2KHSO_4 + 3H_2O$$

$$\underset{\text{chromyl chloride}}{CrO_2Cl_2} + 4NaOH \rightarrow \underset{\text{yellow solution}}{Na_2CrO_4} + 2NaCl + 2H_2O$$

Ans. (B), (C), (D)

2. $Na + (x + y)\,NH_{3(l)} \rightarrow \underset{\text{Solvated cation}}{[Na(NH_3)_x]^+} + \underset{\substack{\text{Solvated electrons}\\ \text{(Blue colour)}}}{(l^\ominus \cdot y(NH_3))^-}$

Due to formation of solvated electron it shows blue colour and electrical conductance exhibits due to both ions. **Ans. (A) & (B)**

3. The reaction of $CH_3—CH=CH—C_6H_4—OH$ with HBr is given as follows.

$$CH_3—CH=CH—C_6H_4—OH + HBr \rightarrow CH_3—CH_2—\underset{\displaystyle Br}{\underset{|}{CH}}—C_6H_4—OH$$

The mechanism of this reaction is represented as follows.

$$CH_3—\overset{\curvearrowleft}{CH}=CH—C_6H_4—OH \xrightarrow[\substack{\text{Slow}\\ \text{Step (1)}}]{H^+} CH_3—CH_2—\overset{+}{CH}—C_6H_4—OH$$

carbonium
(stable due to resonance)

$$\xrightarrow[\text{Fast step II}]{Br^-} CH_3—CH_2—\underset{\displaystyle Br}{\underset{|}{CH}}—C_6H_4—OH$$

Ans. (B)

4. Anilium hydrochloride gives the white precipitates of AgCl with $AgNO_3$.

$$C_6H_5NH_3^+Cl^- + AgNO_3 \rightarrow C_6H_5NH_3^+NO_3^- + AgCl\downarrow$$ **Ans. (C)**

5. The energy of an electron on Bohr orbits of hydrogen atoms is given by the expression.

$$E_n = -\frac{\text{Constant}}{n^2}$$

MATHEMATICS - 1998

PART - A

Directions : *Read questions 1 to 40 carefully and choose from amongst the alternatives given below each question the correct lettered choice(s). A question may have ONE OR MORE correct alternatives. In order to secure any marks for a given question, ALL correct lettered alternative(s) must be chosen.*

1. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals :

(A) $128\,\omega$ (B) $-128\,\omega$

(C) $128\,\omega^2$ (D) $-128\,\omega^2$.

2. Let T_r be the r^{th} term of an A.P., for $r = 1, 2, 3, \ldots$ If for some positive integers m, n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals :

(A) $\frac{1}{mn}$ (B) $\frac{1}{m} + \frac{1}{n}$

(C) 1 (D) 0

3. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is :

(A) at least 30 (B) at most 20

(C) exactly 25 (D) none of the above

4. The diagonals of a parallelogram $PQRS$ are along the lines $x + 3y = 4$ and $6x - 2y = 7$. Then $PQRS$ must be a :

(A) rectangle (B) square

(C) cyclic quadrilateral (D) rhombus.

5. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is :

(A) 0 (B) 1

(C) 3 (D) 4

6. Let $f(x) = x - [x]$, for every real number x, where $[x]$ is the integral part of x. Then $\int_{-1}^{1} f(x)\,dx$ is :

(A) 1 (B) 2

(C) 0 (D) $\frac{1}{2}$

7. If $P = (x, y)$, $F_1 = (3, 0)$, $F_2 = (-3, 0)$ and $16x^2 + 25y^2 = 400$, then $PF_1 + PF_2$ equals :

(A) 8 (B) 6

(C) 10 (D) 12

8. If $P(1, 2)$, $Q(4, 6)$, $R(5, 7)$ and $S(a, b)$ are the vertices of a parallelogram $PQRS$, then :

(A) $a = 2, b = 4$ (B) $a = 3, b = 4$

(C) $a = 2, b = 3$ (D) $a = 3, b = 5$

9. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then :

(A) $\alpha = 1, \beta = -1$ (B) $\alpha = 1, \beta = \pm 1$

(C) $\alpha = -1, \beta = \pm 1$ (D) $\alpha = \pm 1, \beta = 1$

10. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn at random, then the probability that 2 white and 1 black ball will be drawn is :

(A) $\frac{13}{32}$ (B) $\frac{1}{4}$

(C) $\frac{1}{32}$ (D) $\frac{3}{16}$

11. The value of the sum $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals :

(A) i (B) $i - 1$

(C) $-i$ (D) 0

12. The number of values of x where the function $f(x) = \cos x + \cos(\sqrt{2x})$ attains its maximum is :

(A) 0 (B) 1

(C) 2 (D) infinite

13. If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real number x, then the minimum value of f :

(A) does not exist because f is unbounded.

(B) is not attained even though f is bounded

(C) is equal to 1

(D) is equal to -1

14. Number of divisors of the form $4n + 2$ $(n \geq 0)$ of the integer 240 is :

(A) 4 (B) 8

(C) 10 (D) 3

15. $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x - 1)}}{x - 1}$:

(A) exists and it equals $\sqrt{2}$

(B) exists and it equals $-\sqrt{2}$

(C) does not exist because $x - 1 \to 0$

(D) does not exist because left hand limit is not equal to right hand limit

16. If in a triangle PQR, $\sin P$, $\sin Q$, $\sin R$ are in A. P., then :

(A) the altitudes are in A. P. (B) the altitudes are in H. P.

(C) the medians are in G. P. (D) the medians are in A. P.

17. If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals :

(A) $(n-1)\,a_n$ (B) na_n

(C) $\frac{1}{2}\,na_n$ (D) None of the above

18. If the vertices P, Q, R of a triangle PQR are rational points, which of the following points of the triangle PQR is/(are) always rational point(s).

(A) centroid (B) incentre

(C) circumcentre (D) orthocentre

(A rational point is a point both of whose co-ordinates are rational numbers)

19. The number of values of c such that the straight line $y = 4x + c$ touches the curve $\frac{x^2}{4} + y^2 = 1$ is :

(A) 0 (B) 1

(C) 2 (D) infinite.

20. If $x > 1$, $y > 1$, $z > 1$ are in G. P., then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in :

(A) A.P. (B) H.P.

(C) G.P. (D) None of the above

21. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is :

(A) 0 (B) 5

(C) 6 (D) 10

22. The order of the differential equation whose general solution is given by $y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x + C_5}$ where C_1, C_2, C_3, C_4, C_5 are arbitrary constants, is :

(A) 5 (B) 4

(C) 3 (D) 2

23. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin\sqrt{x})^2$, then :

(A) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$ (B) $f(x) = \sin x$, $g(x) = |x|$

(C) $f(x) = x^2$, $g(x) = \sin\sqrt{x}$ (D) f and g cannot be determined

24. Let $A_0\,A_1\,A_2\,A_3\,A_4\,A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments $A_0\,A_1$, A_0A_2 and $A_0\,A_4$ is :

(A) $\frac{3}{4}$ (B) $3\sqrt{3}$

(C) 3 (D) $\frac{3\sqrt{3}}{2}$

25. For three vectors $\vec{u}, \vec{v}, \vec{w}$ which of the following expressions is not equal to any of the remaining three ?

(A) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (B) $(\vec{v} \times \vec{w}) \cdot \vec{u}$

(C) $\vec{v} \cdot (\vec{u} \times \vec{w})$ I(D) $(\vec{u} \times \vec{v}) \cdot \vec{w}$

26. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then the probability that only two tests are needed is :

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

27. Let $h(x) = \min \{x, x^2\}$, for every real number of x. Then :

(A) h is continuous for all x

(B) h is differentiable for all x

(C) $h'(x) = 1$, for all $x > 1$

(D) h is not differentiable at two values of x

28. If $f(x) = 3x - 5$, then $f^{-1}(x)$:

(A) is given by $\frac{1}{3x-5}$

(B) is given by $\frac{x+5}{3}$

(C) does not exist because f is not one-one

(D) does not exist because f is not onto.

29. If $\overline{E}$ and $\overline{F}$ are the complementary events of events E and F respectively and if $0 < P(F) < 1$, then.

(A) $P(E/F) + P(\overline{E}/F) = 1$ (B) $P(E/F) + P(E/\overline{F}) = 1$

(C) $P(\overline{E}/F) + P(E/\overline{F}) = 1$ (D) $P(E/\overline{F}) + P(\overline{E}/\overline{F}) = 1$

30. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then :

(A) $x = 3, y = 1$ (B) $x = 1, y = 3$

(C) $x = 0, y = 3$ (D) $x = 0, y = 0$

31. A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals :

(A) $\frac{1}{2}$ (B) $\frac{1}{32}$

(C) $\frac{31}{32}$ (D) $\frac{1}{5}$

32. An n – digit number is a positive number with exactly n digits. Nine hundred distinct n –digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is :

(A) 6 (B) 7

(C) 8 (D) 9

33. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals :

(A) $\frac{1}{2}$ (B) $\frac{7}{15}$

(C) $\frac{2}{15}$ (D) $\frac{1}{3}$

34. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then :

(A) $b_0 = 1, b_1 = 3$ (B) $b_0 = 0, b_1 = n$

(C) $b_0 = -1, b_1 = n$ (D) $b_0 = 0, b_1 = n^2 - 3n + 3$

35. Which of the following number(s) is/are rational ?

(A) $\sin 15°$ (B) $\cos 15°$

(C) $\sin 15° \cos 15°$ (D) $\sin 15° \cos 75°$

36. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, $S(x_4, y_4)$, then :

(A) $x_1 + x_2 + x_3 + x_4 = 0$ (B) $y_1 + y_2 + y_3 + y_4 = 0$

(C) $x_1 x_2 x_3 x_4 = c^4$ (D) $y_1 y_2 y_3 y_4 = c^4$

37. If E and F are events with $P(E) \leq P(F)$ and $P(E \cap F) > 0$, then :

(A) occurrence of $E \Rightarrow$ occurrence of F

(B) occurrence of $F \Rightarrow$ occurrence of E

(C) non-occurrence of E $\Rightarrow$ non-occurrence of F

(D) none of the above implications holds

38. Which of the following expressions are meaningful question

(A) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (B) $(\vec{u} \cdot \vec{v}) \cdot \vec{w}$

(C) $(\vec{u} \cdot \vec{v})\, \vec{w}$ (D) $\vec{u} \times (\vec{v} \cdot \vec{w})$

39. If $\int_0^x f(t)\, dt = x + \int_x^1 t\, f(t)\, dt$, then the value of $f(1)$ is :

(A) $\frac{1}{2}$ (B) 0

(C) 1 (D) $-\frac{1}{2}$

40. Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number x. Then :

(A) h is increasing whenever f is increasing

(B) h is increasing whenever f is decreasing

(C) h is decreasing whenever f is decreasing

(D) nothing can be said in general.

ANSWERS

1. (D)	**2.** (C)	**3.** (C)	**4.** (D)	**5.** (B)	**6.** (A)
7. (C)	**8.** (C)	**9.** (D)	**10.** (A)	**11.** (B)	**12.** (A)
13. (D)	**14.** (A)	**15.** (D)	**16** (B)	**17.** (C)	**18.** (A)
19. (C)	**20.** (B)	**21.** (C)	**22.** (C)	**23.** (A)	**24.** (C)
25. (C)	**26.** (B)	**27.** (A), (C), (D)	**28.** (B)	**29.** (A), (D)	**30.** (D)
31. (A)	**32.** (B)	**33.** (B)	**34.** (B)	**35.** (C)	
36. (A), (B), (C), (D)		**37.** (D)	**38.** (A), (C)	**39.** (A)	**40.** (A), (C)

SOLUTIONS

1. $(1 + \omega - \omega^2)^7 = (-\omega^2 - \omega^2)^7$

$= (-2\omega^2)^7 = (-2)^7 (\omega^2)^7 = -128 \cdot \omega^{14} = -128\omega^2$

Therefore, (D) is the Ans.

2. Let $T_m = a + (m-1)\, d = \frac{1}{n}$

and $T_n = a + (n-1)\, d = \frac{1}{m}$

$\Rightarrow \quad (m-n)\, d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn} \quad \Rightarrow \quad d = \frac{1}{mn}$

Again $T_{mn} = a + (mn-1)\, d$

$= a + (mn - n + n - 1)\, d$

$= a + (n-1)\, d + (mn - n)\, d$

$= T_n + n(m-1) \cdot \frac{1}{mn}$

$= \frac{1}{m} + \frac{(m-1)}{m} = \frac{1}{m} + 1 - \frac{1}{m} = 1$

Therefore, (C) is the Ans.

3. Let number of newspaper which are read be n. Then

$60n = 300 \times 5$

$\Rightarrow \quad n = 25$

Therefore, (C) is the Ans.

4. Slope of $x + 3y = 4$ is $-1/3$

and slope of $6x - 2y = 7$ is 3.

PHYSICS - 1998

PART - A

Directions

1. *Section I consists of 40 objective type questions.*
2. *This section should take about one hour to answer.*
3. *Each question in this section carries 2 marks.*

1. A transistor is used in common emitter mode as an amplifier, then :

(A) the base emitter junction is forward biased

(B) the base emitter junction is reverse biased

(C) the input signal is connected in series with the voltage applied to bias the base emitter junction

(D) the input signal is connected in series with the voltage applied to bias the base collector junction.

2. Water from a tap emerges vertically downwards with an initial speed of 1.0 m/s. The cross-sectional area of tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is steady, the cross-sectional area of stream 0.15 m below the tap is :

(A) 5.0×10^{-4} m^2 (B) 1.0×10^{-4} m^2

(C) 5.0×10^{-5} m^2 (D) 2.0×10^{-5} m^2

3. A real image of a distant object is formed by a planoconvex lens on its principal axis. Spherical aberration :

(A) is absent

(B) is smaller if the curved surface of the lens faces the object

(C) is smaller if the plane surface of the lens faces the object

(D) is the same whichever side of the lens faces the object.

4. Let $\bar{v}$, v_{rms} and v_p respectively denote the mean speed, root mean square speed and most probable speed of the molecules in an ideal monoatomic gas at absolute temperature T. The mass of a molecule is m. Then :

(A) no molecule can have a energy greater than $\sqrt{2}v_{rms}$

(B) no molecule can have speed less then $v_p/\sqrt{2}$

(C) $v_p < \bar{v} < v_{rms}$

(D) the average kinetic energy of a molecule is $\frac{3}{4} m v_p^2$

5. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300K . The ratio of the average rotational kinetic energy per O_2 molecule to per N_2 molecule is :

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) depends on the moment of inertia of the two molecules.

6. A string of length 0.4 m and mass 10^{-2} Kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time Δt. The minimum value of Δt, which allows constructive interference between successive pulses, is :

(A) 0.05 s (B) 0.10 s

(C) 0.20 s (D) 0.40 s

7. Two particles, each of mass m and charge q, are attached to the two ends of a light rigid rod of length 2R. The rod is rotated at constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is :

(A) $q/2m$ (B) q/m

(C) $2q/m$ (D) $q/\pi m$

8. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence 45°. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value (s) of n from the following :

(A) 1.3 (B) 1.4

(C) 1.5 (D) 1.6

9. Let m_p be the mass of proton, m_n the mass of neutron. M_1 the mass of $^{20}_{10}Ne$ nucleus and M_2 the mass of $^{40}_{20}Ca$ nucleus. Then :

(A) $M_2 = 2M_1$ (B) $M_2 > 2M_1$

(C) $M_2 < 2M_1$ (D) $M_1 < 10(m_n + m_p)$

10. A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is :

(A) 0 (B) $\pi/2$

(C) π (D) 2π

11. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$ where n_1 and n_2 are the principal quantum numbers of two states. Assume the Bohr model to be valid. The time period of the electron in the intial state is eight times that in the final state. The possible values of n_1 and n_2 are :

(A) $n_1 = 4, n_2 = 2$ (B) $n_1 = 8, n_2 = 2$

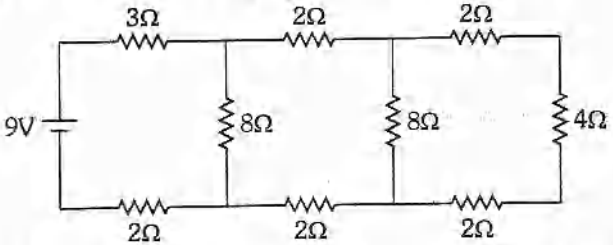
(C) $n_1 = 8, n_2 = 1$ (D) $n_1 = 6, n_2 = 3$

12. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position, where the string is horizontal, is :

(A) $\sqrt{u^2 - 2gL}$ (B) $\sqrt{2gL}$

(C) $\sqrt{u^2 - gL}$ (D) $\sqrt{2(u^2 - gL)}$

13. In the circuit shown in the figure, the current through :



(A) the 3Ω resistor is 0.50 A
(B) the 3Ω resistor is 0.25 A
(C) the 4Ω resistor is 0.50 A
(D) the 4Ω resistor is 0.25 A

14. A dielectric slab of thickness d is inserted in a parellel plate capacitor whose negative plate is at x = 0 and positive plate is at x = 3d. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to 3d :
(A) the magnitude of the electric field remains the same
(B) the direction of the electric field remains the same
(C) the electric potential increases continuously
(D) the electric potential increases at first, then decreases and again increases.

15. The (x, y) coordinates of the corners of a square plate are (0,0), (L, 0), (L, L) and (0, L). The edges of the plate are clamped and transverse standing waves are set up in it. If u (x, y) denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression (s) for u is (are) (a= positive constant) :

(A) a cos ($\pi x/2L$) cos ($\pi y/2L$)
(B) a sin ($\pi x/L$) sin($\pi y/L$)
(C) a sin ($\pi x/L$) sin ($2\pi y/L$)
(D) a cos ($2\pi x/L$) sin ($\pi y/L$)

16. A force $\vec{F} = -K(y\,\hat{i} + x\,\hat{j})$ (where K is a positive constant) acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive x-axis to the point (a, 0) and then parallel to the y-axis to the point (a, a). The total work done by the force F on the particle is :
(A) $-2Ka^2$
(B) $2Ka^2$
(C) $-Ka^2$
(D) Ka^2

17. A small square loop of wire of side l is placed inside a large square loop of wire of side L ($L >> l$). The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to :
(A) l/L
(B) l^2/L
(C) L/l
(D) L^2/l.

18. The half life of ^{131}I is 8 days. Given a sample of ^{131}I at time t = 0, we can assert that :
(A) no nucleus will decay before t = 4 days
(B) no nucleus will decay before t = 8 days

(C) all nuclei will decay before t = 16 days
(D) a given nucleus may decay at any time after t = 0

19. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be ΔP and $1.5\,\Delta P$ respectively. Then :

(A) $4\,m_A = 9m_B$ (B) $2m_A = 3m_B$
(C) $3m_A = 2m_B$ (D) $9m_A = 4m_B$

20. A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is :

(A) $\frac{2}{3}P$ (B) P
(C) $\frac{3}{2}P$ (D) 2P

21. A charge + q is fixed at each of the points $x = x_0, x = 3x_0, x = 5x_0 \ldots \infty$ on the x-axis and a charge –q is fixed at each of the points $x = 2x_0, x = 4x_0, x = 6x_0 \ldots \infty$. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/4\pi\epsilon_0 r$. Then the potential at the origin due to the above system of charges is :

(A) 0 (B) $\frac{q}{8\pi\,\epsilon_0\,x_0 \ln 2}$
(C) ∞ (D) $\frac{q\,\ln(2)}{4\pi\,\epsilon_0\,x_0}$

22. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to :

(A) I (B) $I \sin^2\theta$
(C) $I \cos^2\theta$ (D) $I \cos^2(\theta/2)$

23. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is :

(A) 30 K (B) 18 K
(C) 50 K (D) 42 K

24. A concave mirror is placed on a horizontal table with its axis directed vertically upwards. Let 0 be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be :

(A) real and will remain at C
(B) real and located at a point between C and ∞

(C) virtual and located at a point between C and 0
(D) real and located at a point between C and 0

25. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement (s) from the following :
(A) the entire rod is at the same electric potential
(B) there is an electric field in the rod
(C) the electric potential is highest at the centre of the rod and decreases towards its ends.
(D) the electric potential is lowest at the centre of the rod and increases towards its ends.

26. A positively charged thin metal ring of radius R is fixed in the xy plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is :
(A) periodic for all values of z_0 satisfying $0 < z_0 < \infty$
(B) simple harmonic for all values of z_0 satisfying $0 < z_0 \leq R$
(C) approximately simple harmonic provided $z_0 << R$
(D) such that P crosses 0 and continues to move along the negative z-axis towards $z = -\infty$

27. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth :

(A) The acceleration of S is always directed towards the centre of the earth.
(B) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remain constant.
(C) the total mechanical energy of S varies periodically with time
(D) The linear momentum of S remains constant in magnitude.

28. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A} \times \vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that :
(A) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instants of time.
(B) the component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time.
(C) the magnitude of $\vec{L}$ does not change with time.
(D) $\vec{L}$ does not change with time.

29. During the melting of a slab of ice at 273K at atmospheric pressure :
(A) positive work is done by the ice-water system on the atmosphere.
(B) positive work is done on the ice-water system by the atmosphere.
(C) the internal energy of the ice-water system increases.
(D) the internal energy of the ice-water system decreases.

30. In a p-n junction diode not connected to any circuit :
(A) the potential is the same everywhere
(B) the p-type side is at a higher potential than the n-type side
(C) there is an electric field at the junction directed from the n-side to the p-type side.
(D) there is an electric field at the junction directed from the p-type side to the n-type side.

31. A spherical surface of radius of curvature R, separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O and PO = OQ. The distance PO is equal to :
(A) 5R (B) 3R
(C) 2R (D) 1.5 R

32. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre :
(A) increases as r increases for $r < R$
(B) decreases as r increases for $0 < r < \infty$
(C) decreases as r increases for $R < r < \infty$
(D) is discontinuous at $r = R$

33. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is v/10, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10$ m/s, then λ and f are given by :
(A) $\lambda = 2\pi \times 10^{-2}$ m (B) $\lambda = 10^{-3}$ m
(C) $f = \frac{10^3}{2\pi}$ Hz (D) $f = 10^4$ Hz

34. A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wein constant, $b = 2.88 \times 10^6$ nm-K. Then :
(A) $U_1 = 0$ (B) $U_3 = 0$
(C) $U_1 > U_2$ (D) $U_2 > U_1$

35. Let $[\epsilon_0]$ denote the dimensional formula of the permittivity of the vacuum and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current :
(A) $[\epsilon_0] = [M^{-1}L^{-3}T^2I]$ (B) $[\epsilon_0] = [M^{-1}L^{-3}T^4I^2]$
(C) $[\mu_0] = [MLT^{-2}I^{-2}]$ (D) $[\mu_0] = [ML^2T^{-1}I]$

36. The SI unit of the inductance, the henry can be written as :
(A) Weber/ampere (B) Volt-second/ampere
(C) Joule/(ampere)^2 (D) ohm-second

37. Two very long straight parallel wires carry steady currents I and –I respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is :

(A) $\frac{\mu_0 Iqv}{2\pi d}$ (B) $\frac{\mu_0 Iqv}{\pi d}$

(C) $\frac{2\mu_0 Iqv}{\pi d}$ (D) 0

38. X-rays are produced in an X-ray tube operating at a given accelerating voltage. The wavelength of the continuous X-rays has values from :

(A) 0 to ∞

(B) λ_{min} to ∞ where $\lambda_{min} > 0$

(C) 0 to λ_{max} where $\lambda_{max} < \infty$

(D) λ_{min} to λ_{max} Where $0 < \lambda_{min} < \lambda_{max} < \infty$

39. A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x) = k|x|^3$ where k is a positive constant. If the amplitude of oscillation is a, then its time period T is :

(A) proportional to $1/\sqrt{a}$ (B) independent of a

(C) proportional to $\sqrt{a}$ (D) proportional to $a^{3/2}$

40. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately :

(A) 540 nm (B) 400 nm

(C) 310 nm (D) 220 nm

ANSWERS

1. (A), (C)	**2.** (C)	**3.** (B)	**4.** (C), (D)	**5.** (A)	**6.** (B)
7. (A)	**8.** (C), (D)	**9.** (C), (D)	**10.** (D)	**11.** (A), (D)	**12.** (D)
13. (D)	**14.** (B), (C)	**15.** (B), (C)	**16.** (C)	**17.** (B)	**18.** (D)
19. (C)	**20.** (B)	**21.** (D)	**22.** (A)	**23.** (D)	**24.** (D)
25. (B)	**26.** (A), (C)	**27.** (A)	**28.** (A), (B), (C)		**29.** (B), (C)
30. (C)	**31.** (A)	**32.** (A), (C)	**33.** (A), (C)	**34.** (D)	**35.** (B), (C)
36. (A), (B), (C), (D)		**37.** (D)	**38.** (B)	**39.** (A)	**40.** (C)